Weak Edge-to-Vertex Detour Parameters and their Application to Chronic Pain Networks Modelling

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ABSTRACT:

Let $G = (V, E)$ be a connected simple graph. In this paper, we introduce and investigate the concept of weak edge–vertex detour distance $D_w(e, v)$, defined as the length of a longest admissible $e - v$ path in G. Based on this notion, the weak edge–vertex detour eccentricity of an edge $e \in E$ is defined by $e_2^w(e) = \max_{v \in V} D_w(e, v)$. Using this eccentricity function, we define the weak detour radius $R_2^w(G) = \min_{e \in E} e_2^w(e)$ and the weak detour diameter $D_2^w(G) = \max_{e \in E} e_2^w(e)$, and examine their structural properties. Exact expressions of $e_2^w(e)$, $R_2^w(G)$, and $D_2^w(G)$are obtained for standard graph classes. Further, realization-type results are established for graphs with prescribed weak detour radius and diameter under suitable admissibility conditions. As a practical application, the developed theoretical framework is extended to model anatomical connectivity networks in neuroscience. We focus on **Chronic Pain Networks (CLBP)** to quantify global structural dispersion. We propose a new invariant, the **Weak Detour Pain Index (WDPI)** is defined as, $WDPI(G) = \frac{1}{|E|} \sum_{e \in E} e_2^w\,(e)$. to characterize the topological differences between healthy brain networks and those affected by chronic pain. Our analysis indicates that the WDPI serves as a robust metric for identifying pathological structural alterations, bridging the gap between abstract graph theory and clinical diagnostic modelling.

Keywords: Weak edge–vertex detour distance, Detour eccentricity, Weak detour radius, Weak detour diameter, Graph realization, Path graphs, Pain network.